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CONFIDENCE INTERVALS ON VARIANCE COMPONENTS IN THREE FACTOR CRO--ETC(U)
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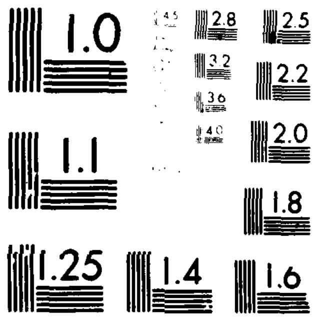

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CONFIDENCE INTERVALS ON VARIANCE COMPONENTS IN THREE FACTOR CROSS-CLASSIFICATION MODELS

by

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ABSTRACT

Consider the three-factor crossed classification components-of-variance model with interaction given by

$$Y_{ijkm} = \mu + A_i + B_j + F_{ij} + C_k + G_{ik} + H_{jk} + P_{ijk} + \varepsilon_{ijkm}.$$

In this paper approximate confidence intervals are exhibited and evaluated for the variances σ_A^2, σ_B^2, σ_C^2. Also a test of H_0: $\sigma_A^2 = 0$ vs. H_a: $\sigma_A^2 > 0$ is given and evaluated.

KEY WORDS: Confidence intervals in variance components models; tests of hypotheses for variance components in 3-way crossed model.

1. Introduction

The problem discussed in this paper is that of setting confidence limits on the variance components in the three-factor crossed components-of-variance model with interaction. The model, described in detail in Sec. 15.5.3 of Graybill (1976), is given by

$$Y_{ijkm} = \mu + A_i + B_j + F_{ij} + C_k + G_{ik} + H_{jk} + P_{ijk} + \epsilon_{ijkm}$$

$$i = 1, 2, \ldots, I > 1; \; j = 1, 2, \ldots, J > 1; \; k = 1, 2, \ldots, K > 1;$$

$$m = 1, 2, \ldots, M > 1.$$

$$E(A_i) = E(B_j) = E(F_{ij}) = E(C_k) = E(G_{ik}) = E(H_{jk}) = E(P_{ijk}) = E(\epsilon_{ijkm}) = 0.$$

$$Var(A_i) = \sigma_A^2, \; Var(B_j) = \sigma_B^2, \; Var(F_{ij}) = \sigma_F^2, \; Var(C_k) = \sigma_C^2,$$

$$Var(G_{ik}) = \sigma_G^2, \; Var(H_{jk}) = \sigma_H^2, \; Var(P_{ijk}) = \sigma_P^2, \; Var(\epsilon_{ijkm}) = \sigma_\epsilon^2.$$

The random variables A_i, B_j, F_{ij}, C_k, G_{ik}, H_{jk}, P_{ijk} and ϵ_{ijkm} are independent and jointly normally distributed.

An analysis of variance (ANOVA) for this model is displayed in Table 1.

The following results for this model are given in Graybill (1976).

(a) $\bar{Y}\ldots$, S_1^2, S_2^2, ..., S_8^2 are complete sufficient (and hence minimal sufficient) statistics for this problem.

(b) $\bar{Y}\ldots$, S_1^2, S_2^2, ..., S_8^2 are jointly independent.

(c) $U_i = S_i^2/\gamma_i^2 = n_i(MSi)/\gamma_i^2$ is distributed as a central chi-square random variable with n_i degrees of freedom for $i = 1, 2, \ldots, 8$.

(d) $\sum_{i=1}^{8} C_i(MSi)$ is the UMVU estimator for $\sum_{i=1}^{8} C_i \gamma_i^2$ for any set of constants $C_1, C_2, \ldots, C_8$.

The problem discussed in this paper is that of obtaining confidence intervals for the variance components σ_A^2, σ_B^2, σ_C^2, σ_F^2, σ_G^2, σ_H^2, σ_P^2, and σ_ε^2.

Variance component models are quite important and useful for many applied problems. Duncan (1974) gives an example where this model can be used. Three analysts each made a determination of the melting point of hydroquinone on each of two days with four different thermometers. The model is a three-way crossed random effect model when the analysts are selected at random from a population of analysts, the thermometers are selected at random from a population of thermometers, and the days are considered a random selection from a population of days. It may be important to determine the variance of the analysts or of the thermometers. Scheffé (1959) also describes a situation when this model is appropriate. The three factors are machines, workers, and batches of material. One point of interest is to determine the variance of the machines or of the workers.

The only variance component of those listed above for which an exact confidence interval exists is σ_ε^2. In this paper an exact (approximate) confidence interval means a confidence interval whose confidence coefficient is equal (approximately equal) to a specified value $1 - \alpha$. Moriguti (1954) gives a procedure that can be used for "good" approximate confidence intervals for σ_F^2, σ_C^2, σ_H^2, and σ_P^2. This procedure was also discussed by Bulmer (1957). Moriguti's procedure applies to the following setup:

1) Let $U_i = n_i V_i / \theta_i^2$ for $i = 1, 2$, be independent chi-square random variables with n_i degrees of freedom.
2) Let $\theta_1^2 = \theta_2^2 + \theta$; i.e., $\theta_1^2 - \theta_2^2 = \theta \geq 0$.
3) If $F_{\alpha:\, n_1,\, n_2}$ is the upper α probability point of Snedecor's F distribution with n_1 degrees of freedom in the numerator and n_2 degrees of freedom in the denominator, then an approximate $1 - \alpha$ upper confidence interval on θ is

$K_\alpha \leq \theta < \infty$ where (1.1)

$$K_\alpha = \frac{V_1}{F_{\alpha:\, n_1,\, \infty}} - V_2 + F_{\alpha:\, n_1,\, n_2}\left(1 - \frac{F_{\alpha:\, n_1,\, n_2}}{F_{\alpha:\, n_1,\, \infty}}\right)\frac{V_2^2}{V_1} \tag{1.2}$$

An approximate $1 - \alpha$ lower confidence interval on θ is $0 \leq \theta \leq K_{1-\alpha}$ where $K_{1-\alpha}$ is obtained from (1.2)by substituting $1 - \alpha$ for α throughout. By substituting the appropriate MSi, MSj, γ_i^2, γ_j^2 from Table 1 for V_i, V_j, θ_i^2, and θ_j^2 respectively one can obtain upper and lower confidence intervals for $\gamma_3^2 - \gamma_7^2 = MK\,\sigma_F^2$ and hence confidence intervals for σ_F^2. Confidence intervals for σ_G^2, σ_H^2, and σ_P^2 can be obtained similarly as the difference of the appropriate γ_i^2 and γ_j^2. The excellence of this procedure is exhibited in Bulmer (1957); also see Scheffé (1959). Howe (1974) gives another method for obtaining confidence intervals for $\theta = \theta_1^2 - \theta_2^2$ when $\theta \geq 0$ but for the problem discussed in this paper we prefer (1.1). None of the methods above gives a confidence interval for σ_A^2, σ_B^2, or σ_C^2. Satterthwaite (1946) and Welch (1956) proposed general procedures for setting approximate $1 - \alpha$ confidence intervals on $\sum C_i \gamma_i^2$, a linear combination of variances. This procedure is useful in certain situations (such as when the C_i are non-negative), but the method is not recommended when some of the C_i are negative.

Since $MJK\,\sigma_A^2 = \gamma_1^2 - \gamma_3^2 - \gamma_5^2 + \gamma_7^2$, these procedures are not recommended for setting confidence limits on σ_A^2. In fact, no "good" method for obtaining approximate $1 - \alpha$ confidence intervals on σ_A^2 has appeared in the literature.

In section 2 formulas are given for approximate upper and lower confidence intervals on σ_A^2 and the approximation is evaluated. Clearly confidence intervals on σ_B^2 and σ_C^2 can be obtained from the formulas for σ_A^2 by substituting the appropriate MSi, γ_i^2, and I, J, K, M; also two-sided approximate confidence intervals can be obtained from the upper and lower confidence intervals. Section 3 contains a discussion of how the formulas in section 2 were obtained, and a summary of the results are given in section 4.

2. Upper and Lower Confidence Intervals on σ_A^2.

The upper approximate $1 - \alpha$ confidence interval on σ_A^2 is $L \le \sigma_A^2 < \infty$ where L is given by

$$L = (MJK)^{-1}\left[\frac{MS1}{F_{\alpha:\, n_1,\, \infty}} - MS5 + F_{\alpha:\, n_1,\, n_5}\left(1 - \frac{F_{\alpha:\, n_1,\, n_5}}{F_{\alpha:\, n_1,\, \infty}}\right)\frac{(MS5)^2}{MS1} - MS3 + F_{\alpha:\, n_1,\, n_3}\left(1 - \frac{F_{\alpha:\, n_1,\, n_3}}{F_{\alpha:\, n_1,\, \infty}}\right)\frac{(MS3)^2}{MS1} + \frac{1}{2}\left(1 + F_{\alpha:\, n_7,\, \infty}\right)MS7\right]. \tag{2.1}$$

If L is negative it is replaced by zero.

The lower approximate $1 - \alpha$ confidence interval on σ_A^2 is $0 \le \sigma_A^2 \le U$ where U is given by

$$U = (MJK)^{-1}\left[\frac{MS1}{F_{1-\alpha:\, n_1,\, \infty}} - MS5 + F_{1-\alpha:\, n_1,\, n_5}\left(1 - \frac{F_{1-\alpha:\, n_1,\, n_5}}{F_{1-\alpha:\, n_1,\, \infty}}\right)\frac{(MS5)^2}{MS1} - MS3 + F_{1-\alpha:\, n_1,\, n_3}\left(1 - \frac{F_{1-\alpha:\, n_1,\, n_3}}{F_{1-\alpha:\, n_1,\, \infty}}\right)\frac{(MS3)^2}{MS1} + \frac{1}{2}\left(1 + F_{1-\alpha:\, n_7,\, \infty}\right)MS7\right]. \tag{2.2}$$

If U is negative it is replaced by zero.

$L \le \sigma_A^2 \le U$ is a two-sided approximate $1 - \alpha$ confidence interval on σ_A^2 when α is replaced throughout with $\alpha/2$.

To evaluate the performances of the upper and lower confidence intervals given above, a simulation study was conducted. We describe only the upper confidence interval $L \le \sigma_A^2 < \infty$ where L is given by (2.1). This interval can be written as

$$MS1\; g\left(\frac{MS3}{MS1}, \frac{MS5}{MS1}, \frac{MS7}{MS1}\right) \le \gamma_1^2 - \gamma_3^2 - \gamma_5^2 + \gamma_7^2 < \infty \tag{2.3}$$

Inequality (2.3) can be rewritten

$$\frac{U_1}{n_1}\, g\left(\frac{\gamma_3^2 U_3}{\gamma_1^2 n_3}\frac{n_1}{U_1}, \frac{\gamma_5^2 U_5 n_1}{\gamma_1^2 U_1 n_5}, \frac{\gamma_7^2 U_7 n_1}{\gamma_1^2 U_1 n_7}\right) \le 1 - \frac{\gamma_3^2}{\gamma_1^2} - \frac{\gamma_5^2}{\gamma_1^2} + \frac{\gamma_7^2}{\gamma_1^2} < \infty$$

where $U_i = n_i\ MSi/\gamma_i^2$ are independent chi-square random variables with n_i degrees of freedom for $i = 1, 3, 5, 7$. Therefore a set of unknown parameters for this problem is given by

$$\frac{\gamma_3^2}{\gamma_1^2}, \frac{\gamma_5^2}{\gamma_1^2}, \frac{\gamma_7^2}{\gamma_1^2} . \tag{2.4}$$

A one-to-one function of the set of unknown parameters given by (2.4) is

$$\theta_F = \frac{M\sigma_F^2}{\sigma_\varepsilon^2 + M\sigma_p^2}, \theta_G = \frac{M\sigma_G^2}{\sigma_\varepsilon^2 + M\sigma_p^2}, \theta_A = \frac{M\sigma_A^2}{\sigma_\varepsilon^2 + M\sigma_p^2} . \tag{2.5}$$

In the simulation study to be described next we fixed γ_7 to be equal to one, since the probabilities involved are invariant under a change of scale in the γ_i's, and considered θ_F, θ_G and θ_A as the unknown parameters. Each of these parameters is allowed to take values from the set

$$\{0.0, 0.01, 0.1, 1.0, 10.0, 100.0\}, \tag{2.6}$$

giving altogether a combination of 216 (6 x 6 x 6) distinct values. Also six set of (IJK) values, shown in Table 2, were investigated. The first step of the simulation was to choose values for I, J, K and α in the model, then the values for the degrees of freedom, n_1, n_3, n_5 and n_7 can be readily computed using formulas in Table 1.

A set of chi-square random variables U_i with respective degrees of freedom n_i, $(i = 1, 3, 5, 7)$ was generated using IMSL program package routine GGCSS in CDC 6400 system. Of the 216 different values for the set of parameters $\{\theta_F, \theta_G, \theta_A\}$, one was chosen and $\gamma_i^2 (i = 1, 3, 5, 7)$ and the parameter of interest $\sigma_A^2 = (\gamma_1^2 - \gamma_3^2 - \gamma_5^2 + \gamma_7^2)/(MJK)$ were calculated. To obtain the values of the mean squares, the obvious transformation $MSi = \gamma_i^2 U_i/n_i$ was used. At this point all the different values necessary to calculate the confidence point were at hand. The procedure was repeated 5000 times and the percentage of times the confidence interval contained the parameter σ_A^2 was calculated for each confidence level. The calculations were repeated with the other 215 values for the set of parameters and the same set of random numbers. The entire procedure was then repeated for different values of (I, J, K). The results are given in Table 2. A simulation study was then conducted for the lower confidence interval on σ_A^2 by repeating the above procedure with the upper confidence point U in (2.2). These results are displayed in Table 3. To demonstrate that Satterthwaite's method is not satisfactory for this problem, a simulation was conducted for that method for $1 - \alpha = .95$. These results are in Table 2.

3. Theory

In this section we describe how the confidence points for σ_A^2 given by (2.1) and (2.2) are obtained. We discuss only the lower confidence point L.

The problem is to determine a function $q^*(Y_{1111}, \ldots, Y_{IJKM})$ such that $P[q^*(Y_{1111}, Y_{1112}, \ldots, Y_{IJKM}) \le \sigma_A^2 < \infty]$ is approximately (and close to) a specified value $1 - \alpha$. The statistics $\bar{Y}\ldots.$, MS1, MS2, ..., MS8 are sufficient, complete (and hence minimal) statistics for this problem so we restrict attention to a function of these for the confidence limits; i.e., we determine a function $q(\bar{Y}\ldots.,$ MS1, MS2, ..., MS8) such that $P[q(\bar{Y}\ldots.,$ MS1, MS2, ..., MS8) $\le \sigma_A^2 < \infty]$ is approximately equal to a specified value $1 - \alpha$. We want the confidence interval to be unchanged if a constant is added to each observation Y_{ijkm}. If this constant is the negative of the value of $\bar{Y}\ldots.$, then MS1, MS2, ..., MS8 are unchanged and $q(\bar{Y}\ldots.,$ MS1, MS2, ..., MS8) becomes q(0, MS1, MS2, ..., MS8). Hence, it is sufficient to look for a function of MS1, MS2, ..., MS8 only for a lower confidence point for σ_A^2. The problem can now be reformulated as follows: Given the jointly independent random variables $U_i = n_i MSi/\gamma_i^2$ for $i = 1, 2, \ldots, 8$ where U_i is a chi-square random variable with n_i degrees of freedom, we want an approximate upper $1 - \alpha$ confidence interval on $\sigma_A^2 = (\gamma_1^2 - \gamma_3^2 - \gamma_5^2 + \gamma_7^2)/MJK$ or equivalently on $MJK\,\sigma_A^2 = \gamma_1^2 - \gamma_3^2 - \gamma_5^2 + \gamma_7^2$. Intuition says that it is sufficient to examine only functions of MS1, MS3, MS5, and MS7 for confidence limits on $MJK\,\sigma_A^2$. The fact that these statistics are inference sufficient for γ_1^2, γ_3^2, γ_5^2, γ_7^2 in the reformulated problem gives justification for this reasoning. For a discussion of inference sufficiency see Fraser (1956) and Rao (1965). The problem now takes the reduced form: Determine a function f(MS1, MS3, MS5, MS7) such that $P[f(MS1, MS3, MS5, MS7) \leq \gamma_1^2 - \gamma_3^2 - \gamma_5^2 + \gamma_7^2]$ is approximately equal to the specified $1 - \alpha$. Now suppose all observations Y_{ijkm} are multiplied by a non-zero constant c. Then MS1, MS3, MS5, and MS7 are multiplied by c^2 and also the uniformly minimum variance unbiased estimator of σ_A^2 is multiplied by c^2. We thus impose the condition that the lower confidence point should be multiplied by c^2 which implies $f(c^2\, MS1,\ c^2\, MS3,\ c^2\, MS5,\ c^2\, MS7) = c^2 f(MS1, MS3, MS5, MS7)$. If we let $c^2 = 1/MS1$ we obtain $f(MS1, MS3, MS5, MS7) = MS1\ f(1, F3, F5, F7) = MS1\ g(F3, F5, F7)$ where $Fi = MSi/MS1$, $i = 3, 5, 7$. So we must determine the function $g(F3, F5, F7)$ such that

$$P[MS1\ g(F3, F5, F7) \leq JKM\ \sigma_A^2]$$

is approximately equal to the specified $1 - \alpha$.

Conditions (1), (2), (3) below seem intuitively desirable to impose on $g(F3, F5, F7)$

(1) The confidence interval is required to reduce to the one given by (1.1) when $\gamma_5^2 = \gamma_7^2 = 0$ and $F5 = F7 = 0$. (Note that $F5 = F7 = 0$ with probability one when $\gamma_5^2 = \gamma_7^2 = 0$.) When $\gamma_5^2 = \gamma_7^2 = 0$ the parameter of interest, $JKM\ \sigma_A^2 = \gamma_1^2 - \gamma_3^2 - \gamma_5^2 + \gamma_7^2$, becomes $\gamma_1^2 - \gamma_3^2$ so replace V_1 by MS1, V_2 by MS3, and n_2 by n_3 in (1.2) and obtain

$$\frac{MS1}{F_{\alpha:\, n_1,\, \infty}} - MS3 + F_{\alpha:\, n_1,\, n_3}\left(1 - \frac{F_{\alpha:\, n_1,\, n_3}}{F_{\alpha:\, n_1,\, \infty}}\right)\frac{(MS3)^2}{MS1} \leq \gamma_1^2 - \gamma_3^2$$

This implies (since $\gamma_5^2 = \gamma_7^2 = 0$ implies $F5 = F7 = 0$ with probability one)

$$MS1\ g(F3, 0, 0) = \frac{MS1}{F_{\alpha:\ n_1,\ \infty}} - MS3 + F_{\alpha:\ n_1,\ n_3} \frac{(MS3)^2}{MS1}\left(1 - \frac{F_{\alpha:\ n_1,\ n_3}}{F_{\alpha:\ n_1,\ \infty}}\right)$$

(2) The confidence interval must be symmetric in MS3 and MS5. This condition is imposed because of the obvious symmetry in the model.

(3) The confidence interval must converge to (1.1) when $K \to \infty$, and when $J \to \infty$.

A natural class of functions that suggests itself for $g(F3, F5, F7)$ is polynomials in F3, F5, and F7. A first approximation is a linear function of the form

$$g(F3, F5, F7) = b_0 + b_1F3 + b_2F5 + b_3F7.$$

This form of function is immediately rejected, because condition (1) and condition (2) on $g(F3, F5, F7)$ require that squared terms in F3 and F5 be included. When this is done the function $g(F3, F5, F7)$ takes the form

$$g(F3, F5, F7) = a_0 + a_1F3 + a_2F5 + a_3F7 + a_4(F3)^2 + a_5(F5)^2$$

where a_0, a_1, a_2, a_3, a_4, and a_5 are constants to be determined so that conditions (1) to (3) are satisfied.

Condition (1) gives

$$a_0 = \frac{1}{F_{\alpha:\ n_1,\ \infty}},\ a_1 = -1 \text{ and } a_4 = F_{\alpha:\ n_1,\ n_3}\left(1 - \frac{F_{\alpha:\ n_1,\ n_3}}{F_{\alpha:\ n_1,\ \infty}}\right).$$

Then condition (2) gives

$$a_2 = -1 \text{ and } a_5 = F_{\alpha:\ n_1,\ n_5}\left(1 - \frac{F_{\alpha:\ n_1,\ n_5}}{F_{\alpha:\ n_1,\ \infty}}\right).$$

Conditions (1), (2) and (3) impose the following limiting conditions on a_3.

$$a_3 \to 1 \text{ as } J \to \infty;\ a_3 \to 1 \text{ as } K \to \infty.$$

There is more than one constant a_3 which will satisfy the restriction $a_3 \to 1$. The following four values for a_3 were chosen to be examined.

(a) $a_3 = 1$

(b) $a_3 = F_{\alpha:\ n_7,\infty}$

(c) $a_3 = 1/F_{\alpha:\ n_7,\infty}$

(d) $a_3 = (1 + F_{\alpha:\ n_7,\infty})/2$.

A preliminary simulation study indicated that $a_3 = (1 + F_{\alpha:\ n_7,\ \infty})/2$ gives better confidence limits than the other three. Thus the final $1-\alpha$ lower confidence point L is given by (2.1). Since σ_A^2 is known to be non-negative, if any confidence point is negative it is replaced by zero. The confidence point given by (2.1) satisfies conditions other than those given above. They are as follows:

(4) The confidence coefficient $\to 1 - \alpha$ when $\sigma_A^2 \to \infty$.

(5) The confidence coefficient $\to 1 - \alpha$ when $J \to \infty$, $K \to \infty$.

(6) The confidence point "coincides" with the parameter as $I \to \infty$. $I \to \infty$ implies that $n_i \to \infty$ for $i = 1, 3, 5, 7$; i.e., sample sizes for all random variables involved tend to infinity. In this limiting case MSi converges in probability to γ_i^2 for $i = 1, 3, 5, 7$, and it is easily seen that the confidence limit given by (2.1) converges in probability to $\gamma_1^2 - \gamma_3^2 - \gamma_5^2 + \gamma_7^2 = \mathrm{MJK}\ \sigma_A^2$.

(7) If σ_F^2 is large relative to the other σ^2s, then MJK $\sigma_A^2 = \gamma_1^2 - \gamma_3^2 - \gamma_5^2 + \gamma_7^2$ is dominated by $\gamma_1^2 - \gamma_3^2$ and S_1^2, S_3^2 tend to be large relative to S_5^2, S_7^2. Thus when S_5^2, S_7^2 are small relative to S_1^2, S_3^2 (actually when $S_5^2 = S_7^2 = 0$) the confidence limit (2.1) reduces to (1.2), a lower confidence limit on $\gamma_1^2 - \gamma_3^2$.

(8) A result similar to (7) applies when σ_G^2 is large.

4. Summary and Conclusions.

By examining Tables 2 and 3 it is quite clear that the lower and upper confidence points given by (2.1) and (2.2) result in confidence coefficients that are quite close to the specified confidence coefficients even for small values of I, J, and K. Thus this procedure can be recommended for confidence intervals on σ_A^2, σ_B^2, and σ_C^2 for the random 3-factor crossed classification model with interaction.

The problem of testing the hypothesis H_0: $\sigma_A^2 = 0$ vs. H_a: $\sigma_A^2 > 0$ in a three factor model with interaction has been discussed by Duncan (1974) and Jeyaratnam (1978). The lower $1 - \alpha$ confidence point L for σ_A^2 given in (2.1) can be used as a test statistic for a test of size α of H_0 vs. H_a. The hypothesis H_0 is rejected if and only if the computed value of the test statistic is positive. For $\alpha = .05$ and $\alpha = .10$ the simulation study described earlier included a tabulation of the proportion of times that L is positive (i.e, that H_0 is rejected) when $\sigma_A^2 = 0$ for values of the parameters θ_F and θ_G given in (2.6). This is the probability of a Type I error and should be close to $\alpha = .05$ and $\alpha = .10$ respectively. The results, given in Table 4, show that the method gives values very close to the specified probability of a Type I error.

Table 1

ANOVA for the Three-Factor Crossed Components-of-Variance Model with Interaction

Source	d.f.	SS	MS	EMS
Total	IJKM			
Mean	1			
A	$I - 1 = n_1$	S_1^2	MS1	$\gamma_1^2 = \sigma_\varepsilon^2 + M\sigma_P^2 + MK\sigma_F^2 + MJ\sigma_G^2 + MJK\sigma_A^2$
B	$J - 1 = n_2$	S_2^2	MS2	$\gamma_2^2 = \sigma_\varepsilon^2 + M\sigma_P^2 + MK\sigma_F^2 + MI\sigma_H^2 + MIK\sigma_B^2$
F	$n_1 \cdot n_2 = n_3$	S_3^2	MS3	$\gamma_3^2 = \sigma_\varepsilon^2 + M\sigma_P^2 + MK\sigma_F^2$
C	$K - 1 = n_4$	S_4^2	MS4	$\gamma_4^2 = \sigma_\varepsilon^2 + M\sigma_P^2 + MJ\sigma_G^2 + MI\sigma_H^2 + MIJ\sigma_C^2$
G	$n_1 \cdot n_4 = n_5$	S_5^2	MS5	$\gamma_5^2 = \sigma_\varepsilon^2 + M\sigma_P^2 + MJ\sigma_G^2$
H	$n_2 \cdot n_4 = n_6$	S_6^2	MS6	$\gamma_6^2 = \sigma_\varepsilon^2 + M\sigma_P^2 + MI\sigma_H^2$
P	$n_1 \cdot n_2 \cdot n_4 = n_7$	S_7^2	MS7	$\gamma_7^2 = \sigma_\varepsilon^2 + M\sigma_P^2$
Error	$IJK(M-1) = n_8$	S_8^2	MS8	$\gamma_8^2 = \sigma_\varepsilon^2$

Table 2

Range* of Confidence Coefficients for Upper Confidence Interval on σ_A^2

			Confidence Interval given by (2.1)			Satterthwaite Procedure
I	J	K	$1-\alpha = .99$	$1-\alpha = .95$	$1-\alpha = .90$	$1-\alpha = .95$
4	4	4	98.84 - 99.62	93.96 - 95.91	89.26 - 91.46	53.66 - 95.18
4	10	4	98.72 - 99.10	94.28 - 95.54	89.98 - 90.32	55.00 - 95.04
4	6	6	98.82 - 99.30	94.18 - 95.36	88.42 - 90.02	55.16 - 95.08
8	4	4	98.36 - 99.18	93.94 - 95.64	88.18 - 90.78	52.80 - 95.32
15	8	8	98.80 - 99.48	94.06 - 95.76	88.90 - 90.64	53.16 - 95.40
20	6	6	98.52 - 99.26	93.70 - 95.42	88.10 - 90.20	52.08 - 95.04

*The entries in the table give the range of confidence coefficients obtained by simulation when the parameters take values from the set given in (2.6).

Table 3

Range* of Confidence Coefficients for Lower Confidence Interval on σ_A^2

I	J	K	$1-\alpha = .99$	$1-\alpha = .95$	$1-\alpha = .90$
4	4	4	98.9 - 99.32	93.96 - 95.66	88.1 - 90.62
4	10	4	98.94 - 99.34	94.02 - 95.64	88.78 - 90.5
4	6	6	98.78 - 99.20	94.14 - 95.20	88.32 - 90.14
8	4	4	98.16 - 99.14	92.38 - 94.90	86.62 - 89.44
15	8	8	98.64 - 99.04	94.12 - 95.18	89.26 - 90.18
20	6	6	98.40 - 99.06	93.66 - 95.43	88.86 - 90.50

*The entries in the table give the range of confidence coefficients obtained by simulation when the nuisance parameters take values from the set given in (2.6).

Table 4

Simulation Probabilities of a Type I Error for Testing $\sigma_A^2 = 0$

(Entries in Columns 6 and 7 are in Percentages)

θ_F	θ_G	I	J	K	$\alpha = 0.05$	$\alpha = .10$
.01	.01	4	4	4	4.40	9.40
		4	10	4	5.28	10.74
		4	6	6	5.14	10.86
		8	4	4	5.72	11.20
.01	1.0	4	4	4	5.08	10.00
		4	10	4	5.28	10.06
		4	6	6	5.18	10.46
		8	4	4	5.36	10.72
.01	100	4	4	4	5.00	9.86
		4	10	4	5.16	9.92
		4	6	6	4.98	10.18
		8	4	4	5.16	10.22
1.0	1.0	4	4	4	4.10	8.92
		4	10	4	4.70	9.92
		4	6	6	4.90	10.32
		8	4	4	4.70	9.92
1	100	4	4	4	4.96	9.88
		4	10	4	5.16	9.96
		4	6	6	4.98	10.20
		8	4	4	5.14	10.20
100	100	4	4	4	4.06	8.56
		4	10	4	4.50	9.84
		4	6	6	4.70	10.14
		8	4	4	4.38	10.58

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ABSTRACT

Consider the three-factor crossed classification components-of-variance model with interaction given by

$$Y_{ijkm} = \mu + A_i + B_j + F_{ij} + C_k + G_{ik} + H_{jk} + P_{ijk} + \epsilon_{ijkm}.$$

In this paper approximate confidence intervals are exhibited and evaluated for the variances σ_A^2, σ_B^2, σ_C^2. Also a test of H_0: $\sigma_A^2 = 0$ vs. H_a: $\sigma_A^2 > 0$ is given and evaluated.